

Quiz 5, MATH 240, Fall 2023

Write your name clearly.

Name:

UID:

- (1) (10 points) Let Ω be a region in $\mathbb{R}^2$ of area 3. If $A = \begin{pmatrix} 2 & 4 \\ 2 & 3 \end{pmatrix}$ is the standard matrix of the linear transformation $T : \mathbb{R}^2 \rightarrow \mathbb{R}^2$, determine the area of $T(\Omega)$ (i.e. the image of Ω under T).

$$\begin{aligned} \text{Area of } T(\Omega) &= |\det(A)| \cdot \text{area}(\Omega) \\ &= |6 - 8| \cdot 3 \\ &= 6. \end{aligned}$$

- (2) (10 points) Find the determinant of $A = \begin{pmatrix} 1 & 1 & 2 & 2 \\ 3 & 5 & 6 & 7 \\ 0 & 0 & 3 & 3 \\ 0 & 0 & -1 & 2 \end{pmatrix}$.

Hint: you can do this without too many cofactors.

$$\begin{aligned} |A| &= \begin{vmatrix} 1 & 1 & 2 & 2 \\ 0 & 2 & 0 & 1 \\ 0 & 0 & 3 & 3 \\ 0 & 0 & -1 & 2 \end{vmatrix} \\ &= \begin{vmatrix} 1 & 1 & 2 & 2 \\ 0 & 2 & 0 & 1 \\ 0 & 0 & 0 & 9 \\ 0 & 0 & -1 & 2 \end{vmatrix} \\ &= - \begin{vmatrix} 1 & 1 & 2 & 2 \\ 0 & 2 & 0 & 1 \\ 0 & 0 & -1 & 2 \\ 0 & 0 & 0 & 9 \end{vmatrix} \\ &= -(1)(1)(-1)(9) \\ &= 9. \end{aligned}$$